

Dynamics of Tachyon and Phantom Field beyond the Inverse Square Potentials

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Abstract

We investigate the cosmological evolution of the tachyon and phantom-tachyon scalar field by considering the potential parameter $\Gamma(= \frac{VV''}{V'^2})$ as a function of another potential parameter $\lambda(= \frac{V'}{\kappa V^{3/2}})$, which correspondingly extends the analysis of the evolution of our universe from two-dimensional autonomous dynamical system to the three-dimension. It allows us to investigate the more general situation where the potential is not restricted to inverse square potential and .One result is that, apart from the inverse square potential, there are a large number of potentials which can give the scaling and dominant solution when the function $\Gamma(\lambda)$ equals 3/2 for one or some values of λ_* as well as the parameter λ_* satisfies condition Eq.(18) or Eq.(19). We also find that for a class of different potentials the dynamics evolution of the universe are actually the same and therefor undistinguishable.

Keywords: Scaling Solution; Dark Energy; three-dimensional autonomous dynamical system; Tachyon Scalar Field.

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1 Introduction

The tachyon field was first arising in the context of string theory [1, 2] and then had been proposed in cosmology to drive the early inflation [3]-[18]. After the important finding of an accelerating expansion of our universe, it became one of the candidates of dark energy to bring on the late time accelerating expansion[19]-[31]. Motivated by the possibility that the equation of state may be less than -1 , its phantom version had also been investigated[32, 33]. Moreover, the cosmological evolution of its quintom version had been considered in literature[34]. For this tachyon-quintom model, the dark energy is composed of the tachyon scalar field as well as the phantom tachyon. These two scalar fields enable the equation of state w to change from $w > -1$ to $w < -1$, just like the quintom model of canonical scalar field. The dynamical evolution of FRW universe filled with a tachyonic fluid plus a barotropic fluid has been extensively studied by performing the phase-plane analysis[27]. However, The potentials of most of the paper about tachyon scalar field are chosen as the inverse square form[19, 20, 25, 27, 28, 29, 33, 43] partly because that only the inverse potential allows constructing a two dimensional autonomous dynamical system using the evolution equations, whereas for any other potentials the number of dimensions will be higher if the system is to remain autonomous[27]. The role of inverse square potential in tachyon scalar field is very similar with the exponential potential in canonical scalar field(quintessence) model[37, 38, 39], where only the exponential potential gives a two dimensional autonomous dynamical system. For the more complicated case that λ is a dynamically changing quantity, i.e. the potential is not the inverse square form, authors had classified any type of tachyon potentials to three classifications and investigated their asymptotic dynamical behavior when λ asymptotically approaches to 0 and $\pm\infty$ [28]. They applied the discussion of constant λ to the case of varying λ and obtained the so-called "instantaneous" critical points. For example, if Γ equals the constant $\Gamma = (n+1)/n$ ($0 < n < 2$), the corresponding potential is the inverse power law potential $V(\phi) = V_0\phi^{-n}$. The critical point P_5 (see the Table 1) will approach to the dark energy scenario in which the universe exhibits an accelerating expansion at late times and the universe in the future will be dominated by the tachyon scalar field since $x(N) \approx \lambda(N)/\sqrt{3} \rightarrow 0$ and $y(N) \approx 1 - \lambda(N)^2/12 \rightarrow 1$ as $\lambda(N) \rightarrow 0$ [28]. Though their classification provided a very useful way to investigate the cosmological evolution for any type of tachyon potentials, their method is numerical and the "instantaneous" critical point is not a true critical point. Here we will provide an exact method to research a large number of potentials beyond the inverse square potentials. This method had been used to investigate the cosmological evolution of the quintessence model with many different potentials as well as the well-known exponential potentials[39] and it had been forwarded to other model[42, 44, 45]. Our method is considering the parameter Γ as a function of λ . In this case, the dynamical system(Eqs.(7-9)) becomes the three-dimensional dynamical autonomous system. Regarding parameter Γ as a function of λ helps us investigate the cosmological evolution with different tachyon potentials exactly. In principle, the potential can be figured out via the relation between parameter Γ and λ , so giving a concrete form of function $\Gamma(\lambda)$ is equivalent to give a concrete form of potential $V(\phi)$. What are the general properties of the critical points when we consider the higher three-dimension autonomous system? Does there still exist scaling solution when we consider other potentials beyond the inverse square potential? Which critical points exist for all tachyon scalar field models and which are only relative to the concrete potentials? In this paper, we will try to respond to these issues. The paper is organized as follows: in Section 2 we present the theoretical framework and give the differential relation between the function $\Gamma(\lambda)$ and potential $V(\phi)$. We find out all the critical points and investigate their properties in Section 3. The cosmological implications of these critical points as well as the summary are presented in section 4.

2 Basic theoretical frame

We start with a spatially flat Friedman-Robertson-Walker universe containing a scalar field ϕ and a barotropic fluid (with state equation $p_b = (\gamma - 1)\rho_b$). For the sake of simplicity and convenience, we present the basic equations directly:

$$p = L(X, \phi) = -V(\phi)\sqrt{1 - \epsilon\dot{\phi}^2} \quad (1)$$

$$\rho = 2L_X X - L(X, \phi) = \frac{V(\phi)}{\sqrt{1 - \epsilon \dot{\phi}^2}} \quad (2)$$

$$H^2 = \frac{\kappa^2}{3} \left[\frac{V(\phi)}{\sqrt{1 - \epsilon \dot{\phi}^2}} + \rho_b \right] \quad (3)$$

$$\dot{H} = -\frac{\kappa^2}{2} \left[\frac{\epsilon \dot{\phi}^2 V(\phi)}{\sqrt{1 - \epsilon \dot{\phi}^2}} + \gamma \rho_b \right] \quad (4)$$

Where $X = \dot{\phi}^2/2$, $L_X = \partial L(X, \phi)/\partial X$. $\epsilon = 1$ for tachyon and -1 for phantom tachyon field. "." means the derivative with respect to time t . The motion equation of the scalar field ϕ is:

$$\ddot{\phi} + 3H(1 - \epsilon \dot{\phi}^2)\dot{\phi} + \frac{\epsilon V'}{V}(1 - \epsilon \dot{\phi}^2) = 0 \quad (5)$$

We defined the dimensionless variables as follows:

$$x = \dot{\phi}, y = \frac{\kappa \sqrt{V}}{\sqrt{3}H}, \lambda = \frac{V'}{\kappa V^{3/2}}, \Gamma = \frac{VV''}{V'^2} \quad (6)$$

Where $V' = dV(\phi)/d\phi$, $V'' = d^2V(\phi)/d\phi^2$. With Eq. (6), Eq. (4-5) can be rewritten in the following dynamical form:

$$\frac{dx}{dN} = -\sqrt{3}(1 - \epsilon x^2)(\sqrt{3}x + \epsilon \lambda y) \quad (7)$$

$$\frac{dy}{dN} = \frac{\sqrt{3}}{2} y (\lambda x y + \frac{\sqrt{3} y^2 (\epsilon x^2 - \gamma)}{\sqrt{1 - \epsilon x^2}} + \sqrt{3} \gamma) \quad (8)$$

$$\frac{d\lambda}{dN} = \sqrt{3} x y \lambda^2 (\Gamma - \frac{3}{2}) \quad (9)$$

where $N = \ln(a)$. The constraint equation from Eq. (3) is:

$$\frac{y^2}{\sqrt{1 - \epsilon x^2}} + \frac{\kappa^2 \rho_b}{3H^2} = 1 \quad (10)$$

If the potential is inverse square potential, λ is a constant and Γ equals $3/2$, then the three dimensional dynamical system Eqs.(7-9) reduces to the two dimensional dynamical autonomous system. If we consider the more complicated case that λ is a dynamically changing quantity ($\Gamma \neq 3/2$), then Eqs.(7-9) is not a autonomous system any more since the parameter Γ is unknown. In this case, we can not analyze the evolution of universe like the inverse square potential exactly. In the paper[28] authors applied the investigation of constant λ to the dynamically changing λ and obtained the so-called "instantaneous" critical points. Here we propose another method, which can analyze the evolution of the universe exactly when the potential is not the inverse square potential. Since λ is the function of tachyon field ϕ and Γ is also the function of ϕ , generally speaking, Γ can be expressed as a function of λ ,

$$f(\lambda) = \Gamma(\lambda) - \frac{3}{2} \quad (11)$$

then Eq.(9) becomes:

$$\frac{d\lambda}{dN} = \sqrt{3} x y \lambda^2 f(\lambda) \quad (12)$$

Hereafter, Eq.(7-8) and Eq.(12) are definitely a dynamical autonomous system. We will show you later that considering Γ as a function of λ can cover many potentials. The three-dimension autonomous system reduces to two-dimension autonomous systems when $f(\lambda) = 0$ (i.e, $\Gamma = 3/2$ and $\lambda = \text{constant}$).

What is the form of the potential if a function of $f(\lambda)$ is given? We start with

$$\frac{d\lambda}{dV} = \frac{d\lambda}{d\phi} \frac{d\phi}{dV} = \frac{1}{V'} \frac{d}{d\phi} \left(\frac{V'}{\kappa V^{3/2}} \right) = \frac{\lambda}{V} f(\lambda) \quad (13)$$

Integrating Eq.(13), we can get the exact function $\lambda(V)$ with respect to V . Then inserting the function $\lambda(V)$ into the definition of λ (Eq.(6)), we obtain following differential equation for potential $V(\phi)$:

$$\frac{dV}{V^{3/2}\lambda(V)} = \kappa d\phi \quad (14)$$

Solving Eq.(14) will give us the expression of potential $V(\phi)$. If $f(\lambda) = 0$, i.e. $\Gamma = \frac{3}{2}$, Eqs.(13-14) will give the potential $V(\phi)$ with the form of $V(\phi) = (\frac{1}{2}\kappa\lambda\phi - c_1)^{-2}$, which is the well-known inverse square potential and has been studied in detail[19, 20, 25, 27, 28, 29, 33, 43]. However, using the method in this paper, there are a number of potentials which can be discussed in principle. Some functions of $f(\lambda)$ and its corresponding potentials $V(\phi)$ are as follows:

$$\begin{aligned} V(\phi) &= \left(\frac{1}{2}\kappa\lambda\phi - c_1\right)^{-2} \text{ for } \Gamma(\lambda) = \frac{3}{2} \\ V(\phi) &= c_2(\phi - c_1)^{\frac{-2}{2n+1}} \quad (n \neq -\frac{1}{2}) \text{ for } \Gamma(\lambda) = n + \frac{3}{2} \\ V(\phi) &= V_0 e^{\alpha\phi} \quad (n = -\frac{1}{2}) \text{ for } \Gamma(\lambda) = n + \frac{3}{2} \\ V(\phi) &= \frac{V_0}{\phi^2 - \phi_0^2} \text{ for } \Gamma(\lambda) = 2(1 - \frac{1}{\kappa^2 V_0 \lambda^2}) \\ \frac{\beta - c_1 V(\phi)^{-\alpha}}{\sqrt{V(\phi)}} &= \frac{1}{2}\kappa\alpha\phi + c_2 \quad (\alpha \neq -\frac{1}{2}) \text{ for } \Gamma(\lambda) = \beta\lambda + (\alpha + \frac{3}{2}) \\ 2\beta V(\phi)^{-\frac{1}{2}} + c_1 \alpha \ln(V(\phi)) &= \kappa\alpha\phi + c_2 \quad (\alpha = -\frac{1}{2}) \text{ for } \Gamma(\lambda) = \beta\lambda + (\alpha + \frac{3}{2}) \\ -\frac{2(-\frac{\beta}{\alpha})^{\frac{1}{n}} \text{Hypergeom}\left(-\frac{1}{n}, \frac{1}{2} \frac{1}{\alpha n}, [1 + \frac{1}{2} \frac{1}{\alpha n}], \frac{c_1 \alpha V(\phi)^{-\alpha n}}{\beta}\right)}{\kappa \sqrt{V(\phi)}} &= \phi + c_2 \text{ for } \Gamma(\lambda) = \beta\lambda^n + (\alpha + \frac{3}{2}) \end{aligned}$$

3 Critical Points and their Properties

The critical points can be found by setting $dx/dN = dy/dN = d\lambda/dN = 0$ and their properties are determined by the eigenvalues of the Jacobi matrix of the three-dimension nonlinear autonomous system Eqs.(7, 8,12). The eigenvalues of each point are obtained by linearizing this nonlinear system around each point. All the points we found are listed in table 1.

Table 1	(λ_c, x_c, y_c)	eigenvalues	Stability
$P_1(\epsilon = 1)$	$\lambda_a, 0, 0$	$-3, 3\gamma/2, 0$	saddle point
$P_2(\epsilon = 1)$	$\lambda_a, \pm 1, 0$	$6, 3\gamma/2, 0$	unstable node
$*P_3(\epsilon = 1)$	$0, 0, \pm 1$	$-3, -3\gamma, 0$	*
$P_4(\epsilon = 1)$	$\lambda_*, \mp \sqrt{\gamma}, \pm \frac{\sqrt{3\gamma}}{\lambda_*}$	$\frac{3}{4}[(\gamma - 2) \pm \sqrt{\mu}], -3\lambda_*\gamma df_*$	Eq.(18)
$P_5(\epsilon = 1)$	$\lambda_*, \mp \frac{\sqrt{3\lambda_*} y_s}{3}, \pm y_s$	$-3 + \frac{\lambda_*^2 y_s^2}{2}, -3\gamma + \lambda_*^2 y_s^2, -\lambda_*^2 y_s^2 \lambda_* df_*$	Eq.(19)
$P_6(\epsilon = -1)$	$\lambda_a, 0, 0$	$-3, 3\gamma/2, 0$	saddle point
$*P_7(\epsilon = -1)$	$0, 0, \pm 1$	$-3, -3\gamma, 0$	*
$P_8(\epsilon = -1)$	$\lambda_*, \pm \frac{\sqrt{3\lambda_*} y_c}{3}, \pm y_c$	$-3 - \frac{\lambda_*^2 y_c^2}{2}, -3\gamma - \lambda_*^2 y_c^2, \lambda_*^2 y_c^2 \lambda_* df_*$	Eq.(20)

* Here one of the eigenvalues of point P_3 and P_7 is zero and the rest eigenvalues are negative, this point is called nonhyperbolic point. the stability of this point cannot be simply determined by the linearization method and need to resort to other method, for instance, the center manifold theorem[40]. The center manifold theorem can help us find the sufficient conditions of stability of the critical systems but it is somehow complicated[39]. Another method is to calculate the three dimensional nonlinear system Eqs.(7,8,12) directly and then plot the phase plane to find the critical point's property numerically.

where λ_a means an arbitrary value of λ and λ_* is the value which makes the function $f(\lambda_*) = 0$, $df_* \equiv \frac{df(\lambda)}{d\lambda}|_{\lambda_*}$.

$$\mu = 17\gamma^2 - 20\gamma + 4 + 48\gamma^2\sqrt{1-\gamma}/\lambda_*^2 \quad (15)$$

$$y_s = \sqrt{\frac{\sqrt{\lambda_*^4 + 36} - \lambda_*^2}{6}}, \quad y_c = \sqrt{\frac{\sqrt{\lambda_*^4 + 36} + \lambda_*^2}{6}} \quad (16)$$

For the point P_4 , we have $0 < \gamma < 1$ since $\Omega_\phi = \frac{3\gamma}{\lambda_*^2\sqrt{1-\gamma}}$. We can also get the condition $\gamma < \frac{\lambda_*^2}{18}(\sqrt{\lambda_*^4 + 36} - \lambda_*^2)$ from $\Omega_\phi < 1$. In fact, Eq.(15) can be rewritten as follows:

$$\mu = (\gamma - 2)^2 - 16\gamma(1 - \gamma)(1 - \Omega_\phi), \quad (17)$$

and then the real part of its eigenvalues $\frac{3}{4}[(\gamma - 2) \pm \sqrt{\mu}]$ are always negative, so the point P_4 will be a stable node or stable spiral(dependent on the sign of μ) when the condition is satisfied below:

$$\gamma < \frac{\lambda_*^2}{18}(\sqrt{\lambda_*^4 + 36} - \lambda_*^2) \text{ and } \lambda_* df_* > 0 \quad (18)$$

For the eigenvalues of P_5 , $\mu_1 = -3 + \frac{\lambda_*^2 y_s^2}{2} = -\frac{3\sqrt{\lambda_*^4 + 36}}{\sqrt{\lambda_*^4 + 36} + \lambda_*^2} < 0$, $\mu_2 = -3\gamma + \lambda_*^2 y_s^2 = 3[\frac{\lambda_*^2}{18}(\sqrt{\lambda_*^4 + 36} - \lambda_*^2) - \gamma]$. So P_5 is a stable node when:

$$\gamma > \frac{\lambda_*^2}{18}(\sqrt{\lambda_*^4 + 36} - \lambda_*^2) \text{ and } \lambda_* df_* > 0 \quad (19)$$

Eq.(18) and Eq.(19) tell us that P_4 and P_5 can never be stable in the same time.

For the eigenvalues of P_8 , $\mu_1 = -3 - \frac{\lambda_*^2 y_s^2}{2} < 0$, $\mu_2 = -3\gamma - \lambda_*^2 y_s^2 < 0$, So P_8 is a stable node when:

$$\lambda_* df_* < 0 \quad (20)$$

The density parameter of tachyon field Ω_ϕ , the equation of state w_ϕ and the decelerating factor q are:

$$\Omega_\phi = \frac{y^2}{\sqrt{1 - \epsilon x^2}} \quad (21)$$

$$\gamma_\phi = 1 + w_\phi = \epsilon x^2 \quad (22)$$

$$q = \frac{3}{2}[(\gamma - \frac{2}{3}) + (\gamma_\phi - \gamma)\Omega_\phi] \quad (23)$$

Other properties of these critical points are listed in table 2.

Table 2	(λ_c, x_c, y_c)	γ_ϕ	Ω_ϕ	q
P_1	$\lambda_a, 0, 0$	0	0	$(3\gamma - 2)/2$
P_2	$\lambda_a, \pm 1, 0$	1	Undefined	-
*P_3	$0, 0, \pm 1$	0	1	-1
P_4	$\lambda_*, \mp\sqrt{\gamma}, \pm\frac{\sqrt{3\gamma}}{\lambda_*}$	γ	$\frac{3\gamma}{\lambda_*^2\sqrt{1-\gamma}}$	$(3\gamma - 2)/2$
P_5	$\lambda_*, \mp\frac{\sqrt{3\lambda_* y_s}}{3}, \pm y_s$	$\frac{\lambda_*^2}{18}[\sqrt{\lambda_*^4 + 36} - \lambda_*^2]$	1	$(3\gamma_\phi - 2)/2$
P_6	$\lambda_a, 0, 0$	0	0	$(3\gamma - 2)/2$
*P_7	$0, 0, \pm 1$	0	1	-1
P_8	$\lambda_*, \pm\frac{\sqrt{3\lambda_* y_c}}{3}, \pm y_c$	$-\frac{\lambda_*^2}{18}[\sqrt{\lambda_*^4 + 36} + \lambda_*^2]$	1	$(3\gamma_\phi - 2)/2$

4 Cosmological Implications

The advantage to investigate the three-dimensional dynamical system is that we can understand the dynamical evolution of universe more deeply, though the process will be more complicated. We find some new critical points which have not been found previously in the two-dimensional system. Another important result is that, besides the inverse square potential, there are many other potentials which also admit the scaling and dominant solutions. Moreover, from the point of view of three-dimensional system, we can find out which critical points exist for all tachyon field and which are only relative to the concrete potentials.

Tachyon Field($\epsilon = 1$): Points P_{1-5} are the whole critical points of tachyon field. Here we do not intend to repeat their properties one by one since the dynamics of tachyon field has been investigated in detail in literatures[27, 28, 41, 42]. However, from the viewpoint of three dimension, we can get some new conclusions. Of all the points, points P_{1-3} are independent of the function $f(\lambda)$ and therefore are nothing to do with the potential $V(\phi)$ while Points P_{4-5} are dependent of the concrete potentials. Points P_4 and P_5 , responding to the scaling and dominant solution, even exist only when the function $f(\lambda)$ can be zero for one or more values of λ_* . For example, there are no P_{4-5} for potential $V(\phi) = c_2(\phi - c_1)^{\frac{-2}{2n+1}}$ ($n \neq 0$). By analyzing three dimension dynamical system, we can study many potentials and get the detailed dynamical evolution of universe. However, our results also show that the properties of most critical points, such as the density parameter Ω_ϕ , the decelerating factor q , are the same for different potentials if they satisfy some conditions. That means the dynamics evolution of universe for a class of different potentials are undistinguishable. Maybe what we need to do is to research a class of potentials, not just one special potential. Reader maybe has found that most critical points and their properties in table 1-2 are the same with inverse square potential[27, 28, 41, 42]. Another result we should emphasize is the point P_3 , which corresponds to the state that universe is dominated by the dark energy with its equation of state w_ϕ being -1 . Moreover, P_3 is a new critical point which does not exist in two dimension system(namely, when potential being inverse square potential). In fact, we find that this critical point is just the "instantaneous" critical point investigated in[28, 41].

Phantom Tachyon Field($\epsilon = -1$): When ϵ equals -1 , Eqs.(7-9) describe the phantom tachyon field with the kinetic term being negative. This is a quite crazy idea but has not been excluded by observations. There are only three critical points(P_{6-8}) for phantom tachyon field. It is quite interesting that all the properties of the point P_6 is the same as point P_1 . Both of them are the saddle points and correspond to the state that our universe is dominated by the barotropic fluid. This fact maybe indicates that, no matter the dark energy is phantom or non-phantom, our universe had truly experienced a stage that dominated by barotropic matter. The properties of P_7 is also the same as P_3 . For point P_7 , the tachyon field behaves as cosmological constant($\gamma_\phi = 0$) and the universe is dominated by dark energy($\Omega_\phi = 1$). As we have pointed out before, they are the nonhyperbolic points and their stability cannot be simply determined by the linearization method. Point P_8 is also a stable solution that phantom dark energy dominated the universe with the equation of state $\gamma_\phi < 0$ and density parameter $\Omega_\phi = 1$. In this case, the universe will evolve to the "big rip" state inevitably.

In summary, in this paper we discuss the three dimensional dynamical autonomous system of tachyon scalar field directly by taking the potential parameter Γ as the function of another potential parameter λ . This is a quite effective method and can be used to investigate the cosmological evolution as long as the parameter Γ can be expressed as the explicit function of λ . From this point of view, the well-known inverse square potential is just a very easy and special case, a large number of potentials can be investigated by this method. When the potential is inverse square potential, the parameter Γ equals $3/2$ and three dimensional autonomous system reduces to the two dimensional autonomous system. We find an important result that, besides the extensively discussed inverse square potential, there are many potentials which can give the tracking solution as long as the function $\Gamma(\lambda) - 3/2 = 0$ for one or several values of λ . Each critical point corresponds to a possible cosmological state of our universe and its stability tells us how our universe evolves to this solution. We find that the existence and properties of some critical points are independent of concrete potentials, so the cosmological solution related to those points are possessed by all tachyon field. For the rest critical points, their existence and properties are related to concrete potentials. In addition, We investigate the phantom tachyon field and find a stable solution corresponding to the equation of state $w_\phi < -1$.

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